

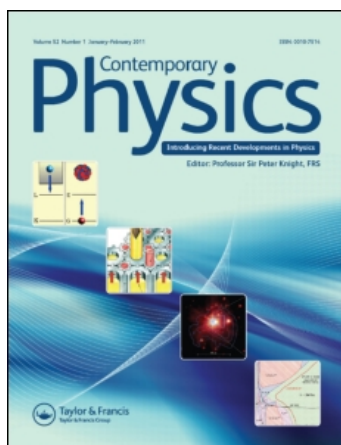
This article was downloaded by: [B-on Consortium - 2007]

On: 18 February 2011

Access details: Access Details: [subscription number 919435512]

Publisher Taylor & Francis

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



## Contemporary Physics

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t713394025>

### Physics of neural synchronisation mediated by stochastic resonance

Lawrence M. Ward<sup>a</sup>

<sup>a</sup> Department of Psychology and the Brain Research Centre, University of British Columbia, Vancouver, BC, Canada

**To cite this Article** Ward, Lawrence M.(2009) 'Physics of neural synchronisation mediated by stochastic resonance', Contemporary Physics, 50: 5, 563 – 574

**To link to this Article:** DOI: 10.1080/00107510902879246

**URL:** <http://dx.doi.org/10.1080/00107510902879246>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

## Physics of neural synchronisation mediated by stochastic resonance

Lawrence M. Ward\*

*Department of Psychology and the Brain Research Centre, University of British Columbia, Vancouver, BC, Canada*

*(Received 3 March 2009; final version received 3 March 2009)*

In this paper I discuss how synchronisation among neurons is affected by noise. Synchronisation of model neurons, which are basically autonomous relaxation oscillators, is enhanced by the addition of low levels of current noise and destroyed by the addition of higher levels of noise. Such neural stochastic resonance also occurs among neural networks in living systems at least at two functionally important scales, both within local networks and between the larger neuronal groups that are thought to implement complex cognitive and perceptual processes.

**Keywords:** neural synchronisation; stochastic resonance; noise; neural networks; perception; cognition

### 1. Introduction

Brains are not mean fields. This complicates a physical description of the brain's dynamics. Although brains are densely interconnected, indeed a brain's neurons probably constitute a connected graph, all structural connections are not functionally relevant at the same time (Figure 1). Moreover, connections among individual neurons are usually one-way, whereas connections between neuronal groups are typically two-way. Complex perceptual and cognitive processes such as seeing, hearing, attention, memory, and consciousness are likely implemented in the brain by networks of neuronal groups, consisting of functionally specialised regions that connect transiently in various subsets depending on the process involved. How these functional connections can form and dissolve over periods of hundreds of milliseconds is one of the central mysteries of neuroscience. One prominent, although still controversial, theory of this mechanism is that neuronal groups are functionally connected when their neurons synchronise at particular oscillation frequencies, allowing for efficient information exchange and maximal mutual influence [1–3]. Although little is yet known about the details of such a process, there is growing evidence that neural synchrony is deeply involved in the brain's operations.

Synchronisation of oscillators is a physical concept that has been studied at least since Huygens discovered that mechanical clocks weakly coupled by being attached to the same wooden board came to tick in precise anti-phase after a while, regardless of their relative phases when started [4]. Fundamentally, neurons are autonomous relaxation oscillators, and

connected groups of neurons in separated brain regions act as autonomous phase oscillators. Thus, physical understanding of synchronous oscillations can be applied to the study of neural synchronisation in the brain. Importantly, noise dramatically affects synchronisation of oscillators, including neural oscillators, enhancing it at low levels and destroying it at high levels, thus displaying the signature of stochastic resonance. In this paper I review basic concepts of synchronisation and stochastic resonance and their application to the problem of understanding how stochastic resonance might modulate, or even mediate, neural synchronisation in the human brain.

### 2. Stochastic resonance

Stochastic resonance was first conceptualised as a way to explain the roughly periodic recurrences of the Earth's Ice Ages [5,6]. In this scenario a very weak sinusoidal oscillation of the Earth's orbit about the sun was magnified by a stochastic resonance with the Sun's noisy energy output into long periods of below normal temperatures followed by long periods of above normal temperatures at the Earth's surface, leading to a series of Ice Ages occurring at roughly 100,000-year intervals. The critical aspect of this system is its nonlinearity: a very small change in total energy flux causes a jump from one state ('warm Earth') to another very different state ('cold Earth') because of a resonance between a stochastic process (the Sun's noisy energy output) and a weak oscillation, whence the name stochastic resonance (henceforth SR). The simplest model of such a dynamic system is a weakly

---

\*Email: lward@psych.ubc.ca

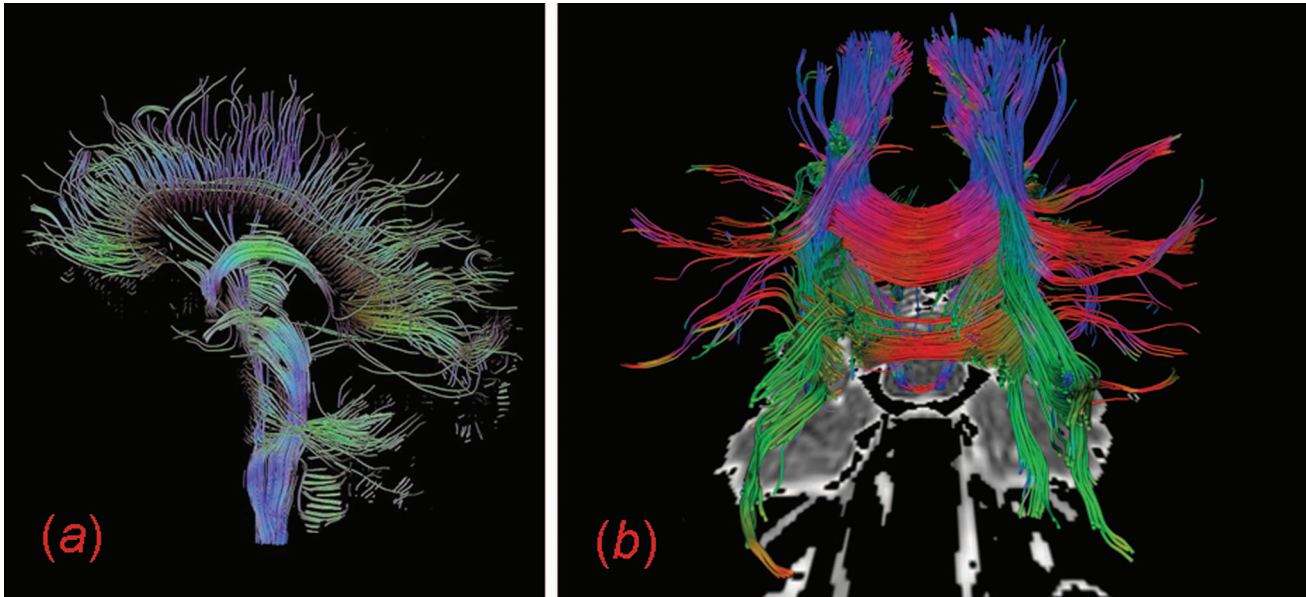


Figure 1. Magnetic resonance diffusion tensor imaging displays of a few of the many connective pathways in the human brain. (a) Sagittal slice through brain and spinal cord (image in public domain at <http://en.wikipedia.org/wiki/Image:DTI-sagittal-fibers.jpg>); (b) pseudo-3D view from posterior (image courtesy of Dr Martin Domin, Ernst-Moritz-Arndt University, Greifswald, Germany, and Dr Lars Linsen, Jacobs University, Bremen, Germany). Colour code indicates 3D directionality of fibre tracts: green indicates fibres running along the anterior-posterior direction, blue indicates the superior-inferior direction, and red indicates the left-right direction.

oscillating two-well potential with random energy additions according to a Gaussian probability distribution (Figure 2). The total energy in the system can be expressed by  $V(t) = a \sin(\omega t) + \varepsilon_\sigma(t)$ , where  $\varepsilon_\sigma(t)$  represents samples of Gaussian noise. The rate of hopping from one well to the other caused by the noise in the absence of the oscillation is expressed by Kramèr's equation:  $r_k = c \exp(-\Delta V/\sigma)$ , where  $\sigma$  is the standard deviation of the noise distribution,  $\Delta V$  is the energy barrier separating the wells, and  $c$  is a constant depending on the particular physical system realising the potential wells. The average time between hops is the inverse of Kramèr's equation,  $T_w = c' \exp(\Delta V/\sigma)$ . When  $T_w = T_\omega/2$ , where  $T_\omega$  is the period of the oscillation, the stochastic hopping process becomes synchronised with the weak oscillation. This and more complicated forms of dynamic SR are ubiquitous in the physical world; many such systems are described and explained in more detail in [7]. Because neurons and neural groups are oscillators they too can engage in dynamic SR. In the present case, however, we are interested in synchronisation between several oscillators mediated by a stochastic process rather than simply that between a stochastic process and an oscillator.

Neurons can also exhibit a simpler form of SR called 'non-dynamical' or 'threshold' SR. Here the critical ingredients are a threshold (generally an energy

barrier of some sort), a weak, sub-threshold signal, and noise. When the noise, the threshold and the signal are all tuned appropriately (Figure 3), the noise induces threshold crossings preferentially at peaks of the signal oscillation [8,9]. In the case of neurons, action potentials (or spikes: brief pulses of cell membrane depolarisation that travel down the axon) are generated in neurons only when a threshold potential around  $-50$  mV at the base of the cell body (soma) is exceeded. This can happen both spontaneously and because of post-synaptic potentials generated by spikes arriving at synapses between a neuron and other neurons. Adding a small amount of random current input to neurons, both peripheral sensory ones and also more central ones, enables them to fire bursts of spikes at the peaks of sub-threshold oscillating inputs (see [9] for a review). Moreover, networks of neurons become synchronised more readily when a small amount of noise is present. In order to understand how this happens, I will first briefly review some concepts of synchronisation and then discuss how synchronisation of neurons is affected by the addition of noise.

### 3. Synchronisation of weakly coupled oscillators

For present purposes, *synchronisation* is taken to be *the adjustment of rhythms of self-sustained oscillating*

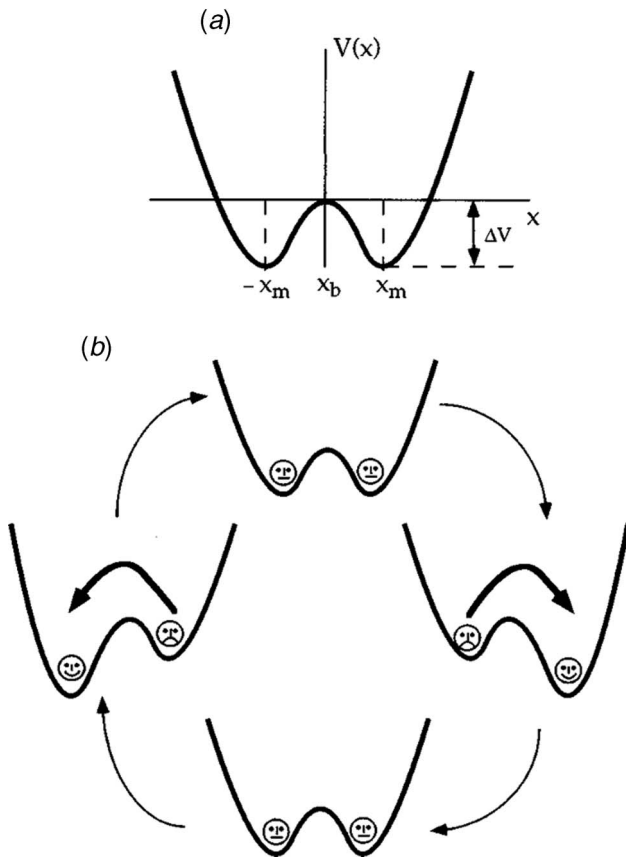


Figure 2. (a) Double well potential with energy barrier  $\Delta V$ . (b) Weak oscillation of potential at frequency  $\omega$  transiently reduces the barrier periodically, making it easier for the system to hop from one well to the other. When  $\omega$  is resonant with the average hopping time from the added random energy kicks, the hopping occurs preferentially in the indicated direction at the indicated times, a dynamic stochastic resonance. Reprinted with permission from [7]. © (1998) by the American Physical Society.

systems caused by their interaction (cf. [4]). The exact state of any self-sustained periodic oscillator can be described precisely by specifying its phase,  $\phi$ , in radians, from 0 to  $2\pi$  [4]. Although phase grows with time it is usually treated as a cyclic variable, that is, it jumps back to zero after it reaches  $2\pi$ , and thus can be visualised as going around a circle. In any situation in which two oscillators maintain a fixed phase difference,  $\phi_1 - \phi_2 = c$ , for some period of time, the oscillators are said to be *synchronised*, even if they are out of phase ( $\phi_1 - \phi_2 \neq 0$ ). When the phases change independently of each other, the oscillators are not synchronised, although they may occasionally have the same phase. *Synchronisation* of oscillators with similar natural frequencies happens when they are weakly coupled for a sufficient duration because each influences the other by perturbing (advancing or retarding) its phase, tending to maintain the phases of the oscillators in a fixed relationship and their oscillation frequencies,

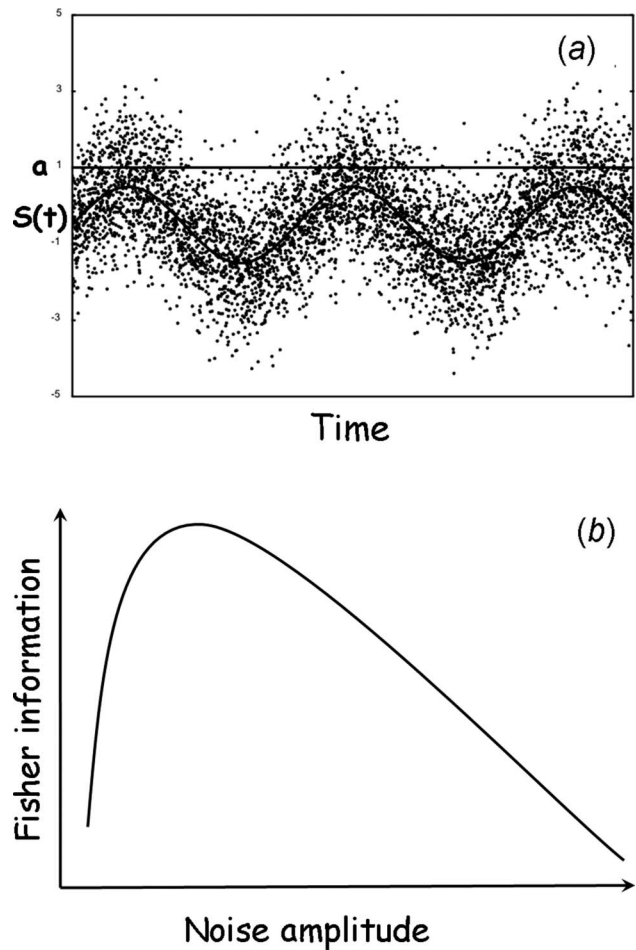


Figure 3. (a) Sub-threshold sinusoidal signal,  $S(t)$ , with optimally-tuned added noise,  $\varepsilon(t)$ . Dots represent  $S(t) + \varepsilon(t)$ ;  $a$  represents the threshold. Notice that  $S(t) + \varepsilon(t) > a$  most often at the peaks of the signal oscillation and least often at the troughs, transmitting the oscillation from the sub-threshold signal into the probability of a supra-threshold event. (b) Fisher information is maximal when noise is optimal. Fisher information is the inverse of the asymptotic variance of any regular estimator; in this case  $S(t)$  is estimated from threshold crossings at each of many discrete time points,  $t$ , by  $\hat{S}(t) = a - F^{-1}(1 - \hat{p})$ , where  $\hat{p}$  is the estimate of the probability of a threshold crossing and  $F^{-1}$  is the inverse of the distribution function associated with the probability density function of the noise [8].

$\omega_1$  and  $\omega_2$ , near a common frequency,  $\omega_0$ . For any given coupling strength, there is a range of natural frequency mismatch over which synchronisation may take place. Figure 4 shows a plot of coupling strength,  $\varepsilon$ , versus natural frequency difference,  $\omega_1 - \omega_2$ , that illustrates this region of synchronisation, called an Arnold tongue, for a range of coupling strengths and an arbitrary oscillator. Very strong coupling, for example the pendula of two clocks connected by a metal bar, results in a single oscillator, and synchronisation is no longer an issue. The boundary between

weak and strong coupling is vague, but the independence of the two systems is the usual criterion: if one system ceases oscillating then the other should not be forced also to cease.

Natural systems are noisy. This means that the parameters of natural oscillators, such as neurons, vary somewhat randomly and thus they cannot achieve exact synchronisation (constant phase difference) for any significant period of time. In particular, the phase of natural oscillators exhibits *phase diffusion*, or random perturbations, because of noise. Fortunately, we can define average parameter values (average period and thus average frequency) over relatively long time periods, and speak of *stochastic phase locking*, where because of phase diffusion we can only identify a tendency for the phase difference to remain bounded within a particular phase interval. Even when we find evidence of stochastic phase locking, however, in a very noisy system the noise will sometimes cause large jumps of  $2\pi$  in phase. These are called *phase slips*. In this simple characterisation noise is a nuisance – it only makes synchronisation more difficult.

Synchronised groups of neurons show some of the characteristics of simple phase oscillators, like pendulum clocks, and their synchronisation can be described as above. Individual neurons, however, are more usefully described as *relaxation oscillators* [10]. Relaxation oscillators display more complicated cycles than simpler oscillators. In particular they display periods of slow and fast motion within a single cycle, making their cycles resemble a sequence of pulses. This is

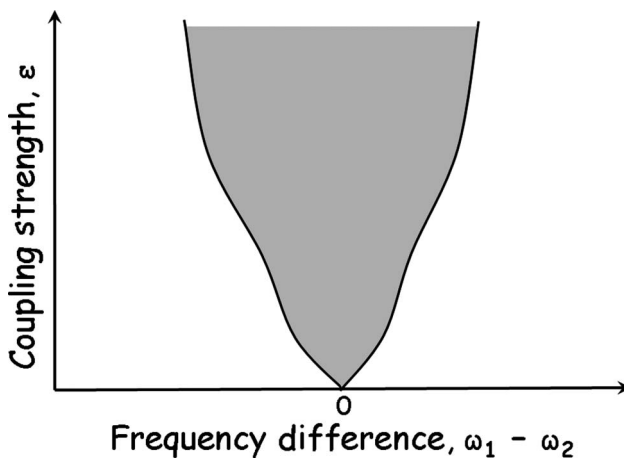


Figure 4. Arnold tongue (or phase-locking region) in grey for two independent, equally-mutually-coupled oscillators. For a natural frequency difference approaching 0 the required coupling strength approaches 0. Also notice that as the coupling strength increases so does the frequency difference over which phase locking can be achieved. Asymmetric coupling yields asymmetric Arnold tongues with the more influential oscillator producing a greater shift toward its natural frequency.

exactly what the rhythm of a neuron looks like, with action potentials, or spikes (fast), separated by periods of slower buildup of potential toward the spike threshold (caused by internal processes and also by post-synaptic potentials). Interactions between coupled neurons is usefully described by the infinitesimal phase response curve (PRC), which depicts the shift in phase,  $\Delta(\phi)$ , and thus the shift in the moment of firing the next spike, of one neuron caused by the reception of a spike (resulting in a post-synaptic potential) from another one as a function of the phase of the receiving neuron [11]:

$$\Delta(\phi) := 2\pi \left( 1 - \frac{\hat{T}(\phi)}{T} \right). \quad (1)$$

In Equation (1), it is assumed that time,  $t$ , is converted to phase,  $\phi$ , by  $\phi = 2\pi t/T$  where  $T$  is the time between spikes, and  $\hat{T}(\phi)$  is the revised time between spikes for a given phase. PRCs can be measured empirically for neurons and also computed from the equations of state in various neuron models. Figure 5 displays an example of a PRC of a generic neuron. Clearly there is a preferred phase, near the end of each free-running cycle just before a spike would naturally occur, at which an incoming spike has the largest effect on the phase of the neuron [12]. Moreover, both positive (phase advance) and negative (phase retardation) effects on phase can occur, depending on whether the PRC is positive or negative when the input occurs. This mechanism of phase perturbation can give rise to

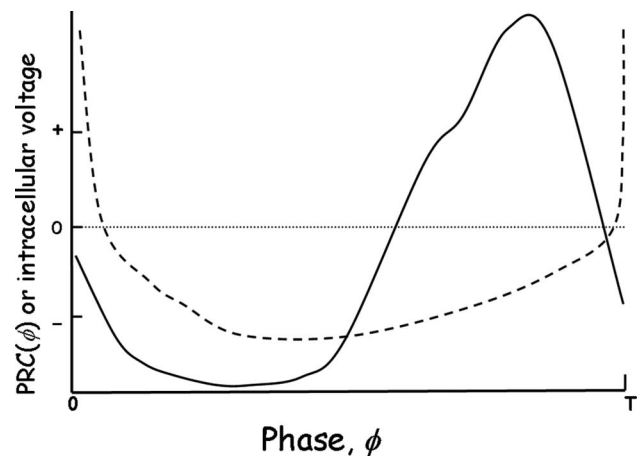


Figure 5. Phase response curve (solid line) for a generic neuron. The dashed line represents the average intracellular voltage during the spike firing cycle, with the spike itself divided between the beginning and the end of the cycle. Only PRC scale is shown; intracellular voltage scale is not shown. Based on a figure in Chapter 10 of [12], Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, MIT Press, 2007.

synchronisation between neurons either by one neuron driving another or by neurons mutually perturbing each other's phase. The Arnold tongues of synchronisation regions for pairs of neurons and neuron models can be computed from their PRCs and their periods (see [12]).

Weak coupling within networks of oscillators whose natural frequencies are fairly close together, including oscillating neurons, gradually brings about deviations of the natural oscillations until a substantial, and possibly ever-changing, group of them is synchronised. This state of affairs was first described by Winfree (e.g. [13]), who also was the first to reduce the equations of state of weakly perturbed oscillators to a phase model, which is simply a differential equation describing the oscillator's change in phase as the result of small perturbations. A canonical phase model for a set of  $n$  weakly coupled oscillators looks like this:

$$\frac{d\phi}{dt} = 1 + \varepsilon \text{PRC}(\phi) h_i(t), \quad (2)$$

where  $h_i(t)$  represents the influence of the  $n-1$  other oscillators on the phase of oscillator  $i$ , with the influence depending on its phase via the PRC. A particularly useful analysis of synchronisation of a fully coupled (mean field) network of oscillators was given by Kuramoto [14] (see also ([12] chapter 10), for a more detailed analysis of this and other models, and [15] for its relation to other important problems). Such analyses begin with the division of the oscillation into two time scales, a fast one that represents the free-running progression of the natural oscillation, and a slow one that represents the collective effects of the rest of the network on the phase:

$$\phi_i(t) = t + \varphi_i, \quad (3)$$

where  $t$  represents the fast scale of the free-running oscillator,  $d\phi_i/dt = 1$ , and  $\varphi_i$  represents the slow-scale phase deviation caused by the network. Kuramoto analysed a special case where the influence of the network on each oscillator is represented by its first Fourier term. This results in a phase model for  $\varphi_i$  that looks like

$$\frac{d\varphi_i}{dt} = \omega_i + Kr \sin(\psi - \varphi_i) \quad (4)$$

where  $K$  is the coupling strength between the oscillators, and  $\psi$  is the population phase, that is, the average phase of any synchronised cluster of oscillators of size  $r$ . The  $\omega_i$  are the natural frequencies of the oscillators, which are randomly distributed over some range in this

model, and which pull each oscillator away from any synchronised cluster. Kuramoto proved that in this model synchronisation depends on the coupling strength relative to the distribution of natural frequencies, with larger synchronised clusters emerging and persisting the stronger the coupling for a given distribution of frequencies. Although not mean fields, it is to be expected that networks of real neural oscillators would display similar behaviour in the sense that they would become synchronised via their mutual coupling and would come to have a population dynamic at a particular population frequency that would reflect their natural and driven oscillation frequencies.

#### 4. Noise-mediated synchronisation

Consider now the behaviour of a group of oscillators to which random noise is added. Pikovskii [16] appears to have been the first to demonstrate that adding the same random pulses to a set of non-interacting oscillators can synchronise their phases. Although his was a special case, more general formulations indicate that his result extends to a broad class of non-interacting limit cycle oscillators and to noise that is only correlated across oscillators [17]. In the case of correlated noise, the synchronising effect increases linearly with the noise correlation. The mechanism by which the synchronisation occurs is similar to that described above for interacting oscillators. Noise inputs disturb the phases of individual oscillators according to their PRCs. If the PRCs are flat, then each noise input simply shifts the phases of all of the oscillators by the same amount, leaving their relationship unchanged. If the PRCs have positive and negative parts, however, as in Figure 4 and for most limit cycle oscillators, then those oscillators for which the input occurs in the positive PRC region, just before spike onset in Figure 4, will have their phases advanced, whereas those for which the input occurs in a negative region of the PRC, after the spike in Figure 4, will have their phases retarded, thus tending to bring the overall group of phases closer together, and thus inducing synchronisation. Once the phases are clustered the noise inputs will have similar effects on all of the oscillators, tending to maintain the synchrony.

Since Pikovskii's pioneering work noise-mediated synchronisation has been studied extensively, and synchronisation of a wide variety of oscillator networks, both coupled and uncoupled, and including networks of neurons and of several different model neurons, has been demonstrated (see [18] for a review). In the case of networks of coupled oscillators there are two sources for synchronising effects: the random noise

(depending on correlation) and the inputs to each oscillator from the others to which it is connected (depending on connection strength). Inputs from either source operate through the PRC and reinforce each other, so that noise-induced synchronisation between coupled oscillators, such as neurons, can be very strong. An instructive example of noise-induced synchronisation in a network of coupled model neurons that also shows SR is the system of locally-coupled Fitzhugh–Nagumo model neurons studied in [19] (see also [20] for a precursor). Each model neuron,  $n$ , obeys

$$\begin{aligned} \varepsilon \frac{du(t,n)}{dt} &= u - \frac{u^3}{3} - w + \gamma \sum_{n'} k[u(t,n') - u(t,n)], \\ \frac{dw(t,n)}{dt} &= u + a(n) + dD^{1/2}\xi(t,n) \end{aligned} \quad (5)$$

where time is dimensionless and scaled so that  $u(t,n)$  and  $w(t,n)$  are fast and slow variables, respectively,  $\varepsilon$  is the ratio of the time scales,  $\gamma$  is the coupling strength,  $a(n)$  is the parameter controlling the excitatory dynamics of the individual neurons,  $\xi(t,n)$  is zero-mean Gaussian noise that is independent across neurons,  $k$  and  $d$  are parameters that depend on the network properties, and  $D$  represents noise strength. For  $a(n) > 1.0$  Equation (5) shows the characteristic occasional spiking exhibited by real neurons. Neiman and colleagues [19] showed that for a sufficiently strong coupling, at an intermediate value of the noise variance the network becomes coherent, not only for locally-coupled nearby neurons but also for distant, not-directly-coupled neurons. For smaller noise intensities the network is correlated only locally, and for larger noise intensities it is incoherent. Figure 6 shows an example of their results. SR of synchronisation is indicated because maximum synchrony occurs in an intermediate noise condition, whereas such synchrony is nonexistent when noise is absent, has only small, local, effects when it is weak, and is destroyed by strong noise.

### 5. Neural synchrony, perception, and cognition

Although synchronisation in neural networks is of interest in its own right, its potential for a functional role in animal, especially human, behaviour is what makes its study exciting. The possibility of such a role was dramatically enhanced by the finding that visual stimuli induced synchronisation at about 40 Hz among the responding neurons in the cat's visual cortex [21]. Since that seminal work, it has become increasingly well documented that gamma-frequency (30–50 Hz) oscillations in mammalian sensory cortex are closely associated with processing of sensory stimuli. For

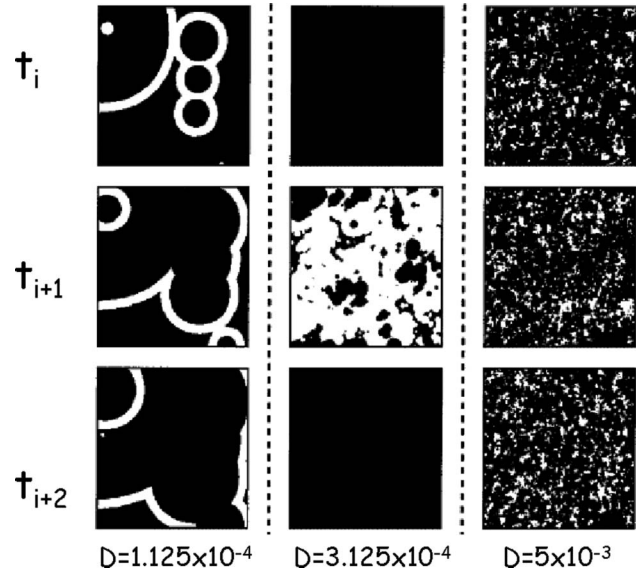


Figure 6. Spiking activity (white pixels, non-spiking neurons are black) in an array of Fitzhugh–Nagumo model neurons at three successive time points (rows from top to bottom). The centre column (intermediate noise level) shows nearly completely synchronous spiking, both local and long-distance, whereas the low noise condition at left shows only local, wavelike synchrony and the high noise condition at right displays a random pattern of spiking. Reprinted with permission from [19]. © (1998) by the American Physical Society.

example, human visual cortex responds in much the same way to the onset of visual stimuli (e.g. [22]), as does human auditory cortex [23]. It has been proposed that these synchronised gamma-frequency oscillations serve to bind together distributed neural representations of environmental stimuli [24]. The gamma-frequency oscillation has also been suggested to underlie perceptual awareness [24,25]. Thus, any process such as SR that modulates such synchronisation could be very important in the understanding of the way in which perceptual and cognitive processes, and even consciousness itself, are implemented in the brain.

Very good progress, too extensive to detail here, is being made in the computational modelling of the role of neural synchronisation in human and other animal cognition. A good example is that of Börgers and colleagues [26] on realistic, coupled neural networks containing both excitatory and inhibitory neurons. It turns out that several different mechanisms, some of them quite subtle, could be involved in the suppression of neural responses to distracting stimuli while attending to a focal stimulus, for example attending to a particular face in a crowd. In their most recent work Börgers and colleagues focus on the fact (e.g. [27]) that neurons in V4 (a visual area in occipital cortex) respond strongly to some visual stimuli at a particular

place on the retina (their receptive field) and more weakly but still positively to others. Interestingly, when both a good and a bad stimulus are present in the receptive field at the same time the overall response is considerably weakened compared to that to the good stimulus alone. But if the animal is induced to pay attention to the good stimulus, then the response is just about the same as if the latter were alone in the receptive field – the weakening attributed to the bad stimulus distractor is suppressed. In the neural network model, which uses realistic (and noisy) models of neurons, each inhibitory and excitatory neuron receives input from all other neurons, although the network would work as well with sparse connectivity as long as each excitatory neuron received input from sufficient inhibitory neurons and vice versa [28]. Each stimulus gives rise to a neural response similar to that in Figure 7(b) when presented alone (to neurons 21–40 as in Figure 7(b), bottom, or to neurons 131–150, not shown), resulting in synchronised gamma-frequency oscillations among all of the inhibitory cells and among the specific subpopulation of excitatory cells stimulated. The response shown in Figure 7(a), however, occurs when both stimuli are presented to the network simultaneously. In this case, the inhibitory

oscillations become desynchronised and the synchronised excitatory response vanishes. Focusing attention on one of the simultaneously presented stimuli is modelled by bathing all neurons in inhibition and also increasing deterministic drive to the stimulated neurons by 30%, thus simulating both inhibitory and excitatory effects of attention. This results in the response shown in Figure 7(b). Synchronised gamma-frequency oscillation is restored among the inhibitory neurons and the response to the attended stimulus emerges in the E-cells as if it were being presented alone. Thus, synchronisation of neurons is closely related to suppression of response to the unattended stimulus as well as to the emergence of a response to the competing attended stimulus, at least in such models.

Neural synchronisation has been associated with a related aspect of attention, orienting to a particular spatial location, using electroencephalographic (EEG) recordings of brain activity and some rather complicated analyses [29]. In these experiments, which are characteristic of modern cognitive neuroscience, human subjects perform carefully controlled tasks while recordings are made of the associated activity in their brains. Doesburg and colleagues [29] had subjects

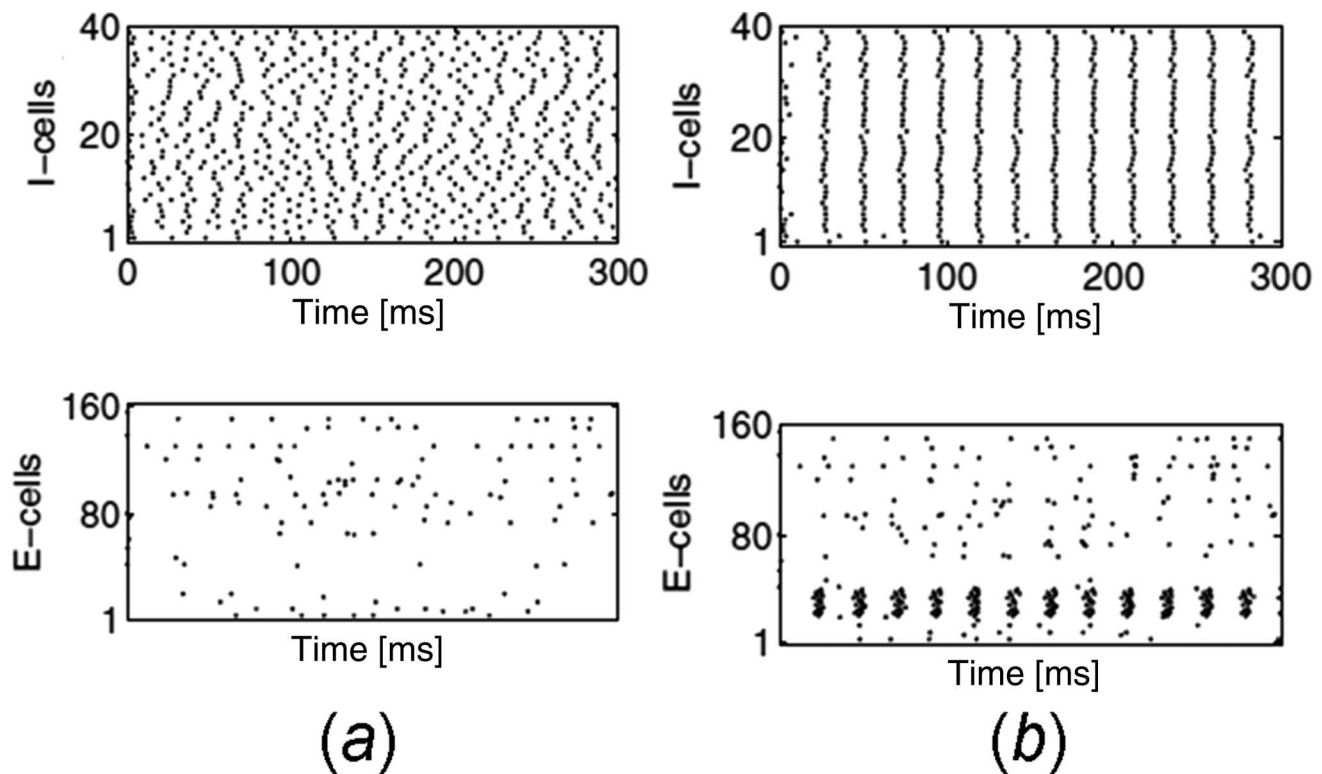


Figure 7. (a) Disrupted synchrony of gamma oscillations leads to weak response to input signal. Dots represent spikes in cell  $\times$  time graph. (b) Synchrony of gamma oscillations is restored, leading to robust response to input signal (bursts of synchronised firing of E-cells 21–40 that received the signal). Reprinted with permission from [26]. © (2008) National Academy of Sciences, USA.



orient their attention, without moving their eyes, to a particular location on a computer monitor in response to a visual cue, at which appeared a subsequent target to which the subjects had to respond. They then analysed synchronisation between the recordings made by various EEG electrodes while subjects were performing this attention orienting task. They ensured that the EEG time series (sampled at 500 Hz) were related to brain regions near the electrode locations on the scalp by first taking the Laplace transform (spatial second derivative) of the data, yielding the scalp current density. They then filtered the broadband time series to produce narrow band signals from which they calculated the analytic signal, defined for a measured function of time,  $f(t)$ , as

$$s(t) = f(t) + i\tilde{f}(t) = A(t)\exp[i\phi(t)], \quad (6)$$

where  $\tilde{f}(t)$  means the Hilbert transform of  $f(t)$ ,

$$\tilde{f}(t) = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{f(\tau)}{t - \tau} d\tau,$$

where  $i = (-1)^{1/2}$  and P.V. indicates the principle value of this improper integral. The Hilbert transform shifts the signal's phase by  $\pi/2$ . For the narrow band signal considered here, the analytic signal is analogous to Euler's identity,  $\exp(i\phi) = \cos\phi + i\sin\phi$ , because  $f(t)$  is sinusoidal and  $\tilde{f}(t)$  is  $f(t)$  phase-shifted by  $\pi/2$ . In Equation (6),  $A(t)$  is the instantaneous envelope amplitude of the signal and  $\phi(t)$  is the instantaneous phase, that is the amplitude and phase at each sample point. Doesburg and colleagues computed an index of

phase locking across trials of the experiment, called the phase locking value, PLV,

$$\text{PLV}_{j,k,t} = N^{-1} \left| \sum_N \exp \{i[\phi_j(t) - \phi_k(t)]\} \right|$$

for each pair of electrodes. PLV ranges between 0, where the phase difference between the signals at the two electrodes varies randomly from trial to trial, to 1, where it is constant across trials. Of course the phase difference is never completely constant in such noisy data, but the reliability of changes in average PLV relative to a baseline can be ascertained using surrogate statistics.

Figure 8 shows some of their results for gamma-band synchronisation. There was a dramatic burst of increased PLV between various electrodes around 300 ms after the cue was presented. This synchronisation was lateralised with respect to the direction in which the cue was directing attention as is shown most clearly in Figure 8(a). Here the right occipital cortex synchronised with other brain regions when the cue oriented attention to the left, and vice versa. Doesburg and colleagues interpreted this burst of lateralised but widespread gamma-band synchronisation as indicating the organisation of an attention network to accomplish the required orienting to the indicated location in visual space. More details about such analyses of synchronisation in whole human brains can be found in [30]. Moreover, substantial work is now being directed at obtaining information about synchronisation between the actual brain regions that generate the scalp signals. This requires

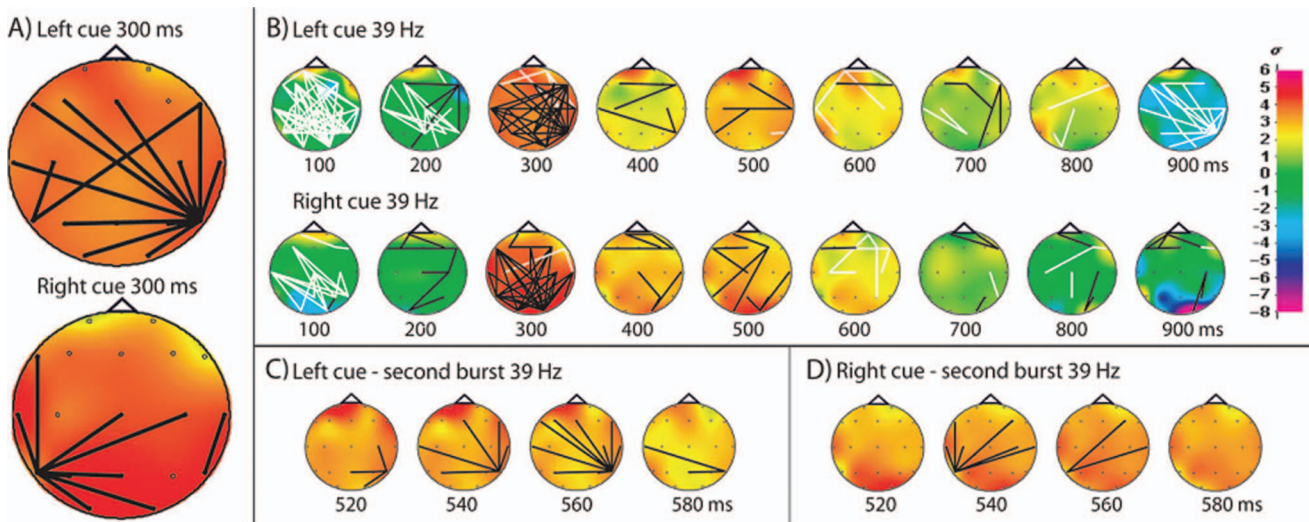


Figure 8. Gamma-band phase locking between occipital cortex and other brain regions involved in attention orienting as a function of time since a cue occurred. Black lines indicate significant increases in phase locking, and white lines indicate decreases, relative to the period before cue onset. Reprinted from [29] with permission. © (2008) by Oxford University Press.

making substantial assumptions about, and modelling of, transmission of electrical and magnetic signals through brain, skull, and scalp tissue, and so is riskier but potentially more informative than the analyses just described [31].

**6. Stochastic-resonance-modulated neural synchrony**

Given the research described so far, it seems plausible both that neural synchronisation plays a role in cognitive processes, and that SR plays a role in neural synchronisation. It remains to put these two ideas together. The first step is to demonstrate noise-induced synchronisation in a real neural system, and an excellent example is provided by the work of Mori and Kai [32]. They recorded the electroencephalogram (EEG) from the occipital cortex (visual) region of the human skull while driving the closed right eye with sub-threshold light flickering at 5 Hz (about 1/2 the frequency of alpha, an endogenous brain rhythm generated in occipital cortex), and the closed left eye with randomly flickering (15 to 60 Hz) light at various intensities. In this design the noise and the sub-threshold stimulus mix in the visual system subsequent

to stimulating the separate visual receptors. Mori and Kai measured spectral power in the EEG at 10 Hz, which indexes local entrainment, that is synchronisation, of 10 Hz neural oscillations to the sub-harmonic 5 Hz driving stimulus (which remember was also sub-threshold). The critical result was that spectral power at 10 Hz was maximal at a non-zero level of noise, demonstrating SR in the human brain. Kitajo and colleagues used a similar design to show that a sub-threshold visual stimulus could also drive overt behaviour, leading to the conjecture that the behavioural driving occurred via noise-induced neural synchronisation [33]. This conjecture was indirectly tested by Kitajo and colleagues in another similar experiment in which EEG was recorded while human subjects detected near-threshold visual stimuli [34]. Global phase locking of signals across all EEG electrodes in three different frequency bands was greatest in the same, optimal noise, condition in which performance was maximised, implying that the conjecture was correct (Figure 9).

A somewhat more direct test of the involvement of SR in neural synchronisation relevant to cognition was attempted by Ward and colleagues [35]. In their

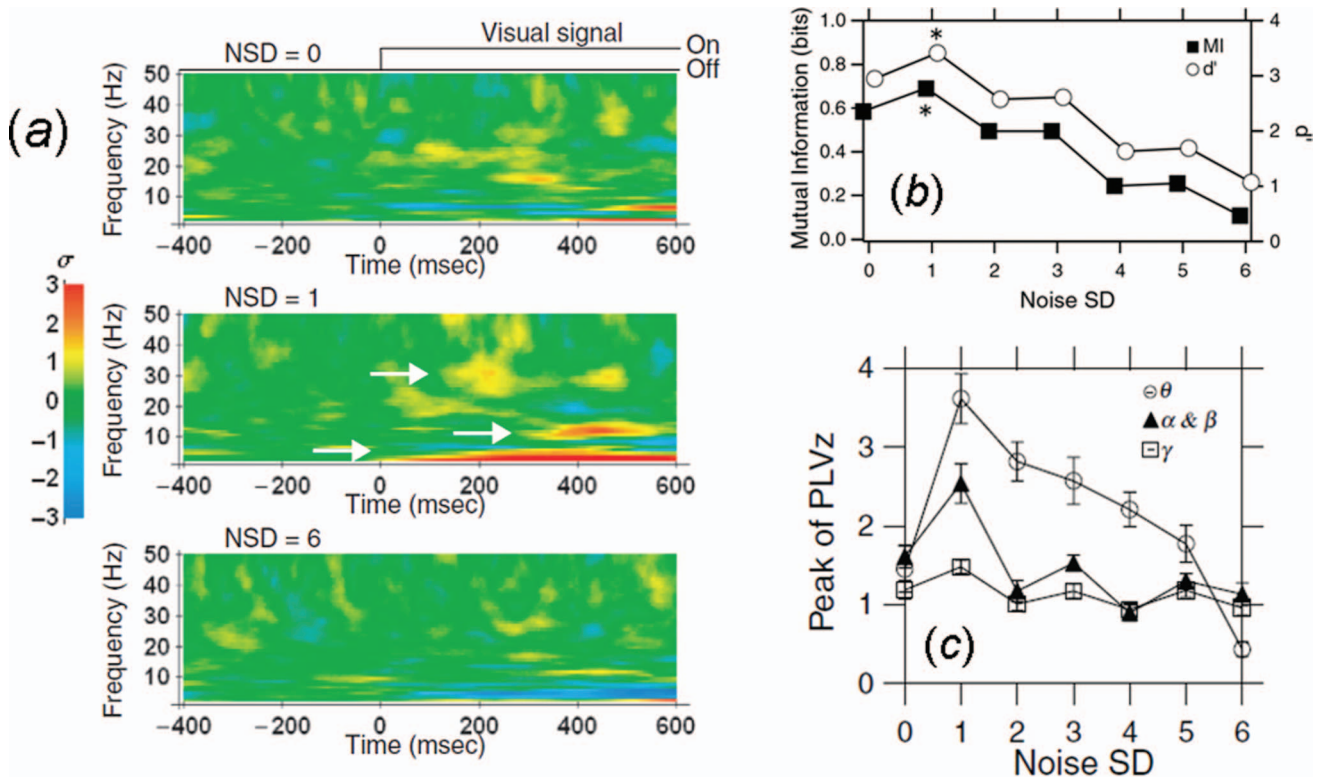


Figure 9. Neural synchrony across all EEG electrodes [(a) and (c)] is greatest for the same non-zero noise condition in which performance was maximal (b). NSD and Noise SD refer to the noise intensity, the standard deviation of the Gaussian noise distribution. The colour scale is in standard deviation units of PLV relative to a pre-stimulus baseline. Reprinted with permission from [34]. © (2007) by the Institute of Physics.

experiment, EEG recordings were made while subjects listened to two streams of near-threshold, brief, tonal auditory stimuli, one to each ear. In each ear occasional stimuli were more intense, and subjects had to push a button whenever the more intense stimuli occurred in the left ear (only). Auditory stimuli of supra-threshold amplitude normally elicit a synchronised response at around 40 Hz lasting about 100–200 ms (or 4–8 cycles) from neurons in the auditory cortex (just above the ear). The near-threshold stimuli used in this experiment, however, would not be expected to elicit such a response. Indeed, this paradigm is often used to test the hearing ability of people for whom making a verbal or other response is not possible, for example young infants or people who are unconscious or suspected of malingering. In this experiment, however, in addition to the near-threshold

tonal stimuli broadband auditory noise was also presented to the left ear, at intensities ranging from 0 (no noise) to about 20 dB above the detection threshold (Sensation Level, or SL) for the noise. Figure 10 shows some preliminary results from this experiment that indicate that, as expected, there was no 40 Hz response to the near-threshold stimuli in the absence of added noise. For a small amount of added noise, however, the transient 40 Hz response appeared, indicating that the noise apparently caused increased neural synchrony in response to the tone. A similar result was obtained for near-threshold stimuli presented to the right ear in the absence of noise to that ear, indicating that the left ear noise combined in the brain with the right ear stimuli to produce a response. This is functionally relevant neural synchrony, because the 40 Hz response indicates neural activity to

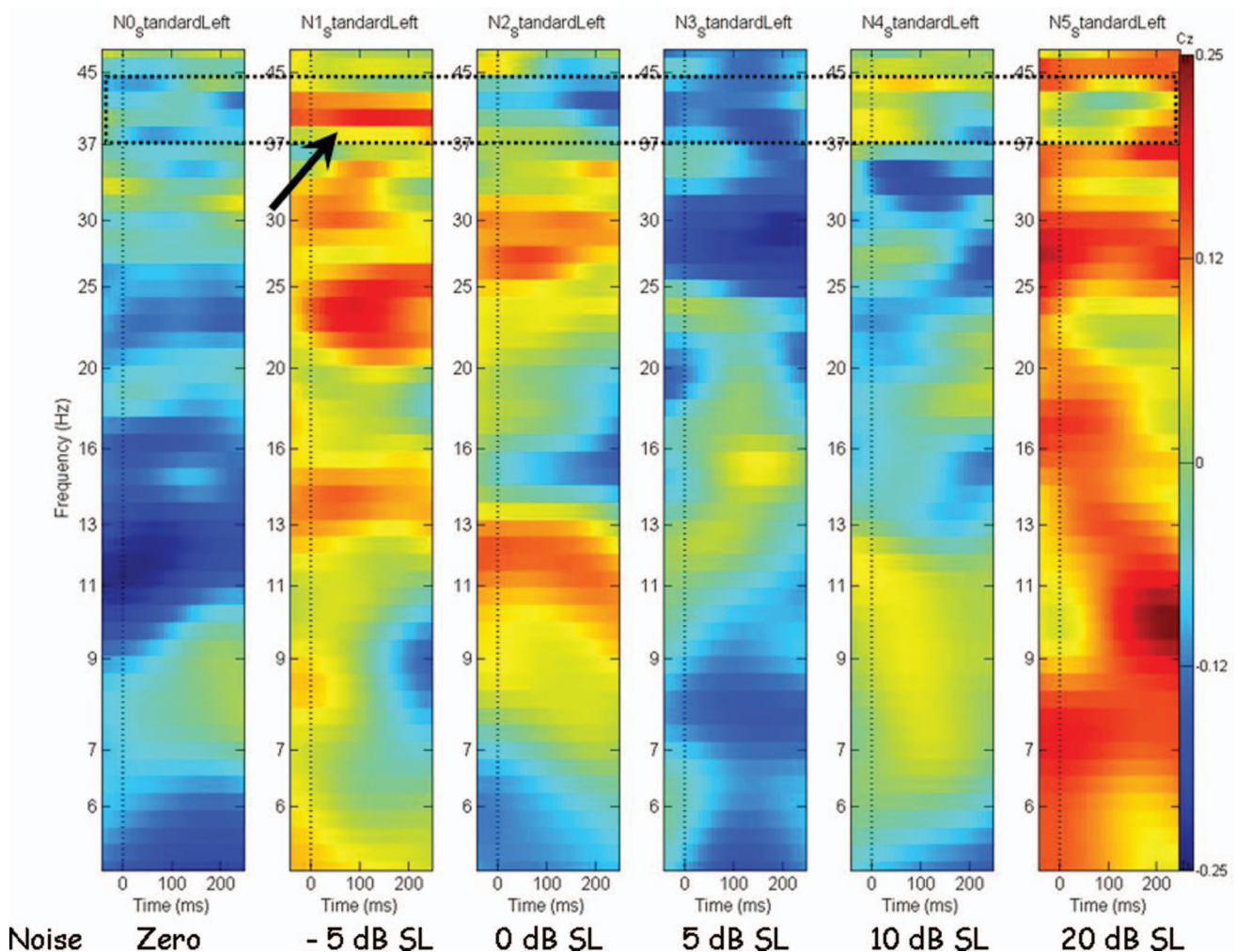


Figure 10. Transient 40 Hz response to near-threshold stimuli is greatest in a non-zero noise condition. Arrow points to the 40 Hz response in the noise = -5 dB SL condition. Colour code represents dB of spectral power relative to that in a 1000 ms baseline immediately prior to stimulus onset [35].

a detectable auditory stimulus. Thus, studies like this one are beginning to make the case for a functionally relevant modulation of neural synchrony by SR.

## 7. Conclusions

I have described evidence that noise plays a role in the neural synchronisation that is associated with perceptual and cognitive processes. It seems that random fluctuations can be helpful under some conditions, as maintained by proponents of SR, and that one of the arenas in which this role might be important is that of neural networks, including those in the brain. This story is far from complete, however. One especially troublesome missing part is a definitive demonstration that the noise that is *intrinsic* to networks of neurons, indeed to any real system, can play a constructive role, or that any living systems were in some way optimised with respect to the ubiquitous noise in which they evolved and that they themselves create. Interestingly, calculations show that the foraging strategies of the zooplankton *Daphnia* might have evolved to generate an optimally noisy distribution of turning angles [36]. Also, preliminary attempts to measure intrinsic noise in the human brain, and to relate its level to performance, have had some success (e.g. [37,38]). We can expect that such efforts will continue and that many of the missing parts will be filled in, or the hypothesis rejected, over the next several years. Even if intrinsic noise and evolution play no significant part in noise-mediated neural synchrony, however, external noise can clearly play a role, and possibly lead to useful applications in prosthetics and other areas.

## Acknowledgements

This work was supported by a Discovery Grant from the Natural Sciences and Engineering Research Council (NSERC) of Canada. I am grateful to my co-authors for their contributions to the work from my lab described herein.

## Notes on contributor

Lawrence M. Ward is a Professor of Psychology and a member of the Brain Research Centre at the University of British Columbia in Vancouver, Canada. He was born in 1944 in Canton, Ohio, USA. He received an A.B. from Harvard University in 1966 and a Ph.D. in Experimental Psychology from Duke University in 1971, where he also studied mathematics. He has always been interested in dynamical systems theory, and did most of his early work in the dynamics of psychophysical judgment and attention. His interest in stochastic resonance and neural synchrony began when he was part of a group that received a grant from the Peter Wall Institute for Advanced Study at UBC from 1996–2000 to study Crisis Points, extreme non-linearities in dynamical systems. Several visits by Frank Moss, a major early contributor to the theory and biological applications of SR, cemented this interest and stimulated both experimental

and theoretical work on SR. This work converged with neural synchrony through the 3-year stay at his lab of Keiichi Kitajo, a young and prolific Japanese researcher, and the work of Kitajo's mentor Yoshiharu Yamamoto. Ward's main interests at present are in the subject matter of this paper as well as in the discovery of the synchronous neural networks that underlie cognitive processes, especially including attention and consciousness.

## References

- [1] F. Varela, J.-P. Lachaux, E. Rodriguez, and J. Martinerie, *The brainweb: phase synchronization and large-scale integration*, Nature Rev: Neurosci 2 (2001), pp. 229–239.
- [2] L.M. Ward, *Synchronous neural oscillations and cognitive processes*, Trends Cog. Sci. 17 (2003), pp. 553–559.
- [3] P. Fries, *A mechanism for cognitive dynamics: neuronal communication through neuronal coherence*, Trends Cog. Sci. 9 (2005), pp. 474–479.
- [4] A.S. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Science*, Cambridge University Press, Cambridge, 2001.
- [5] R. Benzi, A. Sutera, and A. Vulpiani, *The mechanism of stochastic resonance*, J. Phys. A 14 (1981), pp. L453–L457.
- [6] C. Nicolis, *Solar variability and stochastic effects on climate*, Solar Phys. 74 (1981), pp. 473–478.
- [7] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, *Stochastic resonance*, Rev. Mod. Phys. 70 (1998), pp. 223–288.
- [8] P. Greenwood, U.U. Müller, L.M. Ward, and W. Wefelmeyer, *Statistical analysis of stochastic resonance in a thresholded detector*, Austrian J. Stat. 32 (2003), pp. 49–70.
- [9] F. Moss, L.M. Ward, and W.G. Sannita, *Stochastic resonance and sensory information processing: a tutorial and review of application*, Clin. Neurophysiol. 115 (2004), pp. 267–281.
- [10] R. Fitzhugh, *Impulses and physiological states in theoretical models of nerve membrane*, Biophys. J. 16 (1961), pp. 445–466.
- [11] B.S. Gutkin, G.B. Ermentrout, and A.B. Reyes, *Phase response curves give the responses of neurons to transient inputs*, J. Neurophysiol. 94 (2005), pp. 1623–1635.
- [12] E. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, MIT Press, Cambridge, MA, 2007.
- [13] A. Winfree, *Biological rhythms and the behavior of populations of coupled oscillators*, J. Theoret. Biol. 16 (1967), pp. 15–42.
- [14] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*, Springer Verlag, New York, 1984.
- [15] S.H. Strogatz, *Norbert Wiener's brain waves*, in *Frontiers in Mathematical Biology, Lecture Notes in Biomathematics*, S. Levin, ed., Vol. 100, Springer-Verlag, New York, 1994, pp. 122–138.
- [16] A.S. Pikovskii, *Synchronization and stochastization of array of self-excited oscillators by external noise*, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 27 (1984), pp. 390–395.
- [17] J.-N. Teramae and D. Tanaka, *Robustness of the noise-induced synchronization in a general class of limit-cycle oscillators*, Phys. Rev. Lett. 93 (2004), pp. 204103-1–204103-4.

- [18] G.B. Ermentrout, R.F. Galán, and N.N. Urban, *Reliability, synchrony, and noise*, Trends Neurosci. 31 (2008), pp. 428–434.
- [19] A. Nieman, L. Shmansky-Geier, A. Cornell-Bell, and F. Moss, *Noise-enhanced phase synchronization in excitable media*, Phys. Rev. Lett. 83 (1999), pp. 4893–4897.
- [20] P. Jung and G. Mayer-Kress, *Spatiotemporal stochastic resonance in excitable media*, Phys. Rev. Lett. 74 (1995), pp. 2130–2133.
- [21] C.M. Gray and W. Singer, *Stimulus-specific neuronal oscillations in orientation columns of cat visual cortex*, Proc. Natl. Acad. Sci. USA 86 (1989), pp. 1698–1702.
- [22] C. Tallon-Baudry, O. Bertrand, C. Delpuech, and J. Pernier, *Stimulus specificity of phase-locked and non-phase-locked 40 Hz visual responses in human*, J. Neurosci. 16 (1996), pp. 4240–4249.
- [23] C. Pantev, S. Makeig, M. Hoke, R. Galambos, S. Hampson, and C. Gallenii, *Human auditory evoked gamma-band magnetic fields*, Proc. Natl. Acad. Sci. USA 88 (1991), pp. 8996–9000.
- [24] A. Engel and W. Singer, *Temporal binding and the neural correlates of sensory awareness*, Trends Cog. Sci. 5 (2001), pp. 16–25.
- [25] F. Crick and C. Koch, *Some reflections on visual awareness*, Cold Spring Harb. Symp. Quant. Biol. 55 (1990), pp. 953–962.
- [26] C. Börgers, S. Epstein, and N.J. Kopell, *Gamma oscillations mediate stimulus competition and attentional selection in a cortical network model*, Proc. Natl. Acad. Sci. USA 105 (2008), pp. 18023–18028.
- [27] J.H. Reynolds, L. Chelazzi, and R. Desimone, *Competitive mechanisms subserve attention in macaque areas V2 and V4*, J. Neurosci. 19 (1999), pp. 1736–1753.
- [28] C. Börgers and N.J. Kopell, *Synchronization in networks of excitatory and inhibitory neurons with sparse, random connectivity*, Neural Comp. 15 (2003), pp. 509–539.
- [29] S.M. Doesburg, A.B. Roggeveen, K. Kitajo, and L.M. Ward, *Large-scale gamma-band phase synchronization and selective attention*, Cereb. Cortex 18 (2008), pp. 386–396.
- [30] L.M. Ward and S.M. Doesburg, *Synchronization analysis in EEG and MEG*, in *Advances in ERP, EEG, and MEG Analysis*, T.C. Handy, ed., MIT Press, Cambridge, MA, in press.
- [31] S.M. Doesburg and L.M. Ward, *Synchronization between sources: emerging methods for understanding large-scale functional networks in the human brain*, in *Coordinated Activity in the Brain: Measurements and Relevance to Brain Function*, J.-L. Perez-Velazquez, and R. Wennberg, eds., Springer, New York, in press.
- [32] T. Mori and S. Kai, *Noise-induced entrainment and stochastic resonance in human brain waves*, Phys. Rev. Lett. 88 (2002), pp. 218101-1–4.
- [33] K. Kitajo, D. Nozaki, L.M. Ward, and Y. Yamamoto, *Behavioral stochastic resonance within the human brain*, Phys. Rev. Lett. 90 (2003), pp. 218103-1–4.
- [34] K. Kitajo, S.M. Doesburg, K. Yamanaka, D. Nozaki, L.M. Ward, and Y. Yamamoto, *Noise-driven large-scale synchronization of human brain activity enhances perception*, Europhys. Lett. 80 (2007), pp. 400091–400096.
- [35] L.M. Ward, S. Maclean, H. Ueda, K. Kitajo, and A. Kirschner, *Stochastic-resonance mediated neural synchrony in the human brain*, Invited Talk at SR2008 in Perugia, Italy, August, 2008.
- [36] N.D. Dees, S. Bahar, and F. Moss, *Stochastic resonance and the evolution of Daphnia foraging strategy*, Phys. Biol. 5 (2008), pp. 044001-1–6.
- [37] L. Emberson, K. Kitajo, and L.M. Ward, *Endogenous neural noise and stochastic resonance*, in *Noise and Fluctuations in Biological, Biophysical, and Biomedical Systems*, S. Bezrukov, ed., Proceedings SPIE 6602, SPIE, Bellingham, WA, 2007, pp. 6602OT1–6602OT12.
- [38] T. Aihara, K. Kitajo, D. Nozaki, and Y. Yamamoto, *Internal noise determines external stochastic resonance in visual perception*, Vis. Res. 48 (2008), pp. 1569–1573.