Proposed Metameric Indices for Goniochromatic Objects

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Abstract: As a means of determining the degree of metamerism for goniochromatic objects, a generalized illuminant metamerism index for changes in spectral composition of illuminants has been formulated based on a modified LABD index. This index takes into account factors such as color memory and could be used under paramerism conditions. Consequently, a geometric metamerism and a flop index is also presented. Nevertheless, their estimation is based on the establishment of goniochromatic discrimination functions. All these concepts could be useful in many industrial-coating applications to achieve an overall measure of variation in goniochromatism on metallic materials. Col Res Appl, 27, 382–390, 2002; DOI 10.1002/col.10092

Key words: metamerism; generalized tristimulus values; goniochromatic discrimination functions; goniochromatism; metallic materials

INTRODUCTION

For many industrial color reproduction processes, metamerism is one of most important undesirable effects in an object's color appearance. This concept is defined in terms of a pair of stimuli and distinguishes between illuminant metamerism (when a pair of samples present the same tristimulus values but different spectral radiant–power distributions, under the same viewing conditions[1–3]), and observer metamerism (when a pair of samples match for some observers but not for others, under the same viewing conditions, due differences in the color-matching functions[4,5]). In most common situations, an effective measurement could be made for these phenomena, where reflecting materials do not depend heavily on the angular conditions of illumination and viewing; thus, spectral reflectance curves could be defined only as a function of the wavelength. For these, International Commission on Illumination (CIE) tristimulus values may be measured and, consequently, metameric indices for changes in spectral composition of illuminants and for change in observer have been defined.[1,2,5–10] Nevertheless, since the 1950s, the coating industry has extensively used special pigments such as metal-flake or pearlescent pigments,[11] which have simulated and produced new, attractive effects in many applications (e.g., automotive coatings) such as metallic and nacreous luster or goniochromism, but have at the same time introduced new situations such as geometric metamerism. For these objects, reflectance curves strongly depend on the angular conditions of illumination and viewing, and goniochromatism arises.[12,13]

A general index of illuminant metamerism should consider these situations, taking into account uniformly colored objects, and thus CIE tristimulus values, as a limit. At the same time, this general index should be independent of the change in the spectral composition of illuminants, and paramerism conditions should be taken into account.[1,4] On the other hand, key factors in color matching,[3] such as high luminance levels, are not considered in traditional metameric indices. In this way, color memory and chromatic adaptation are also important factors that should be taken into account to achieve a better correlation with visual perception.[7,10]

My work seeks to enhance these concepts by introducing generalized tristimulus values by means of goniochromatic discrimination functions. These expressions can be useful to provide an overall measure of goniochromatism. Based on these definitions, a generalized illuminant metamerism index for changes in spectral composition of illuminants, a geometric metameric index, and a flop index are presented.

METHODS

Generalized Tristimulus Values

The appearance of metallic materials changes when the angle of illumination or the angle of view shifts with respect

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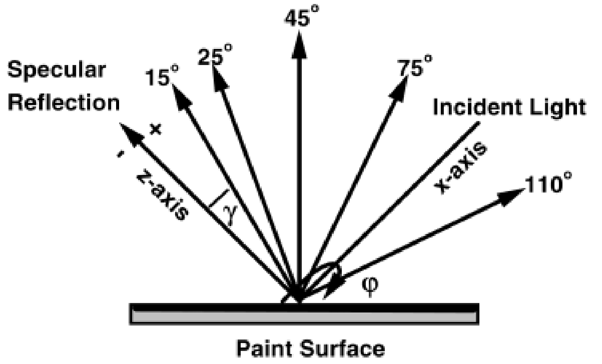

FIG. 1. Schematic representation of a typical five-angle spectrophotometer. Reference *z* and *x* axes are transformed to the specular and incident light-beam direction, respectively. The angle of illumination is fixed at the standard geometry of 45° from normal.

to the colored surface without changes in light source or observer. Therefore, these gonioapparent effects could be estimated in a general way if the reflectance factor could be described with the use of spherical coordinates, and if the spectral reflectance were integrated along the solid angle. For the evaluation of goniochromatism, multiangle spectrophotometers have been designed for repeated measures, placing an illuminating source at a fixed illumination angle of 45° from normal and locating detectors in multiple directions above or below the source axis.[12,14,15] For this arrangement, a transformation of coordinates would be suitable. The z axis that defines the polar coordinate, and the x axis rotates to match the specular reflection and illuminating direction, respectively, so that $R = R(\lambda, r, \psi, \theta)$, where r, ψ, and θ represent the radial, azimuth, and polar coordinates, respectively, in relation to this coordinate system. This change enables us to redefine the polar angle θ as the aspecular angle[12] or the effect angle[15] γ, defined as the angle between specular reflection and the observational direction. This angle is more appropriate for measuring goniochromatism, because specular reflection is avoided while evaluating the reflectance spectrum along a solid angle.[12] This avoids the problems of coatings appearing very light and glaring in this direction and of color-matching breaking down at high luminance levels.[3] Figure 1 represents a traditional, five-angle spectrophotometer with the source direction fixed at 45° from normal and five detectors place at the aspecular angles of $\gamma = 15°, 25°, 45°, 75°$, and $110°$, respectively.

Radial dependency could be simplified, assuming that variations due to the measuring distance in colorimetric instruments or the viewing distance from a few meters,[12] could be disregarded. On the other hand, if we consider a homogeneous coating medium in which pigments are well dispersed, and if we ignore border effects, then the reflectance could also be considered constant along the azimuthal coordinate ψ. Therefore, R could be simplified to $R = R(\lambda, \gamma)$, a generalized tristimulus values $\mathbf{T}_i$ (in bold type) could be defined for large visual fields as

$$\mathbf{T}_i = \kappa_i \iint \Phi_i \cdot R(\lambda, \gamma) \cdot \bar{x}_{i10}(\lambda) \cdot \mathrm{d}\lambda \cdot \mathrm{d}\Omega, \quad i = 1, 2, 3, \qquad (1)$$

where κ_i is a constant, $R(\lambda,\gamma)$ is the reflectance spectrum, $\bar{x}_{i10}(\lambda)$ the CIE 1964, 10° color-matching function associated with the ith generalized tristimulus value for a large visual field, Φ_i represents a new function that defines human goniochromatic sensibility to perceived goniochromatism, and $\mathrm{d}\Omega$ is the differential solid angle [$\mathrm{d}\Omega = \sin(\gamma)\mathrm{d}\psi\mathrm{d}\gamma$, from the reference system represented in Fig. 1 $\gamma \in (-\pi/4, 3\pi/4)$, $\psi \in (0, \pi)$]. When there is no angular dependency, R is only a function of the wavelength, and tristimulus values could be defined for a fixed aspecular angle of 45°. In this case, the tristimulus values calculated are equivalent to those resulting from the CIE 45/0 measuring geometry on diffuse materials.[16] Therefore, κ_i values can be determined if generalized tristimulus values $\mathbf{T}_i$, match with CIE tristimulus values X_i (without bold type), when there is no goniochromatism. For this situation, $R(\lambda,\gamma) = R(\lambda)$ if we integrate first with respect to λ:

$$\mathbf{T}_i = X_i \cdot \kappa_i \int \Phi_i \cdot \mathrm{d}\Omega = X_i \Leftrightarrow \kappa_i = \frac{1}{\int \Phi_i \cdot \mathrm{d}\Omega}, \quad i = 1, 2, 3. \qquad (2)$$

Eq. (2) defines the *CIE limit* if, and only if, Φ_i are pure geometrical functions [i.e., $\Phi_i = \Phi_i(\gamma,\psi)$], or, assuming objects with homogeneous coatings, $\Phi_i = \Phi_i(\gamma)$. These goniochromatic functions represent the human color-discrimination capacity according to the viewing direction with respect to a geometric reference direction, situated in this case on the specular reflection. It is noteworthy that Eq. (2) implies materials with perfect diffuse reflectance, but even uniformly colored objects could present a slightly goniochromatic variation due partly to local irregularities, the heating of the sample, or the influence of instrumental variables such as the spectrophotometric short-term reproducibility.[14] Therefore, Eq. (2) represents a desirable asymptotic limit. Given that multiangle spectrophotometers measures at a few given angles and wavelengths, we can express the generalized tristimulus values in their discrete form as

$$\mathbf{T}_i = \kappa_i \cdot \pi \cdot \sum_{\lambda} \sum_{\gamma} \Phi_i(\gamma) \cdot R(\lambda, \gamma) \cdot \overline{x_{i10}}(\lambda) \cdot \sin(\gamma) \cdot \Delta\lambda \cdot \Delta\gamma, \quad i = 1, 2, 3, \qquad (3)$$

with

$$\kappa_i = \frac{1}{\pi \cdot \sum_{\gamma} \Phi_i(\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}, \quad i = 1, 2, 3, \qquad (4)$$

where π represents the azimuthal contribution and $\Delta\gamma$ is expressed in radians. If we add with respect to λ, replace each κ_i following Eq. (4) and transform each tristimulus value at each aspecular angle γ to the CIE (1976) L*a*b* space, the generalized coordinates **L***, **a*,** and **b*** could be written as

$$\mathbf{L^*} = \frac{\sum_{\gamma} \Phi_L \cdot (\gamma) \cdot L^*(\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}{\sum_{\gamma} \Phi_L \cdot (\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}, \qquad (5)$$

$$\mathbf{a^*} = \frac{\sum_{\gamma} \Phi_a \cdot (\gamma) \cdot a^*(\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}{\sum_{\gamma} \Phi_a \cdot (\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}, \qquad (6)$$

$$\mathbf{b^*} = \frac{\sum_{\gamma} \Phi_b \cdot (\gamma) \cdot b^*(\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}{\sum_{\gamma} \Phi_b \cdot (\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}, \qquad (7)$$

where $L^*(\gamma)$, $a^*(\gamma)$, $b^*(\gamma)$ represents the luminance, red–green and yellow–blue color coordinates for a specific aspecular angle γ, respectively. The range of aspecular angles needed to characterize gonioappearance depends on the pigments involved and the visual correlation with colorimetric data from multiangle measurements. In most common cases, at least three measuring geometries are usually adequate, taking on the range of near-specular, face, and flop angles, respectively.[12,17] The Deutsches Institut für Normung (DIN) recommends[15] the use of three aspecular angles of 25°,45°,75°, and optionally 110°, whereas the American Society for Testing Materials (ASTM) prescribes[16] three aspecular angles of 15°,45°, and 110°.

A Generalized Metamerism Index for Changes in Spectral Composition of Illuminants

The CIE recommends use of the special metamerism index for changes in illuminant CIE 1973 for a pair of conventional specimens. This signifies that the degree of metamerism for illuminant changes could be ascertained when an exact match is made for the first illuminant; thus, the metamerism index (M_t) is the color difference found for the second illuminant. When there is no exact match with respect to the first illuminant, the CIE advises taking this failure into account.[1,2] Several mathematical corrections have been used.[1,9,10] Alternatively, other indices naming general indices of metamerism[6,8,10] have been proposed. These general indices are based on the spectral differences of a pair of samples and thus avoid illuminant influence. Nevertheless, they are found to have no visual correlation.[4,10] Similarly, specific corrections from the first illuminant, such as subtractive or multiplicative correction,[1,3] do not improve visual correlation.[10] On the other hand, attempts to evaluate metameric differences from color-constancy indices should be improved.[10]

Visual detection of metameric pairs normally uses comparisons by means of simultaneous and successive color matching. When a pair of juxtaposed color samples match or nearly match for a specific illuminant condition, the same pair is tested for a second illuminant, both illuminant situations being compared by memory. Subtle factors govern this procedure, notably, chromatic adaptation and color memory. Conventional light booths provide chromatic adaptation during color matching where the color surround constitutes a major factor.[18,19] On the other hand, color memory is the basis of successive color matching when the influence of the delay time and regions in the color space become crucial.[20,21] To take some of these factors into account in a plausible way, we prefer to adopt the metameric index CIE 1976 ΔL^*,Δa^*,Δb^* differences (LABD),[10] which correlated well with visual perception in dyed objects.[10] Use of the LABD index consists of defining variations on each member of a color-sample pair under two different illuminant conditions, which represents a chromatic adaptation from the first illuminant for each sample of the pair, and is compatible with the fact that chromatic discrimination is poorer at high levels of chromatic adaptation.[22] According to the commutative law of addition, each factor of the LABD index could be rewritten in terms of variations from the reference to the specimen panel, in the first and second illuminant condition. In this form, it takes into account paramerism conditions. Moreover, color-sample differences in the second illuminant are compared to those found under the first illuminant condition; thus, this also expresses a short-term color-memory form.

Despite the fact that CIELAB color coordinates are scaled in terms of illuminant color coordinates from the XYZ transformation, CIELAB color space is not illuminant-uniform in terms of chromatic adaptation.[7] A normalized Berns and Billmeyer formula[7] take this factor into account, using the nonlinear transformation proposed by Nayatani *et al.* (cited by Berns and Billmeyer[7]) and this formula could also be used on the basis of the good correlation with visual perception,[10] but multiplicative corrections have a less clear meaning in terms of a color-memory interpretation. On the other hand, the Nayatani *et al.* transformation has not been demonstrated in goniochromatic samples. As a preliminary attempt, we would scale each color-coordinate variation of the LABD index in terms of the corresponding illuminant color-coordinate variations for each pairwise illuminant comparison, but illuminant color coordinates are not defined in the CIELAB color space. Nevertheless, illuminant influence could take into account normalizing each member of LABD metameric index by means of the corresponding variations in color-surround coordinates under the two illuminant tested, thus representing a form of adaptive color shift:

$$\Delta E_{ji} = \left[\sum_k \left(\frac{(\Delta X_k)_j - (\Delta X_k)_i}{\Delta' X_k^S} \right)^2 \right]^{1/2},$$

$$\text{where, } \Delta X_k,\ \Delta' X_k^S \in [\Delta L^*,\ \Delta a^*,\ \Delta b^*,\ \Delta C^*,\ \Delta H^*], \qquad (8)$$

where ΔX_k and $\Delta' X_k^S$ denote the kth color-coordinate variation between the reference and specimen panels calculated in the jth and ith illuminant condition (subscripts j,i) and the variation in the kth color-surround coordinate between the two illuminant tested, respectively. For the calculation of $\Delta' X_K^S$, black and nonuniform surrounds should be excluded. Nevertheless, many commercial light booths are usually painted light gray, in order to improve the correlation between visual observations in the booth and in sunlight.[12] Therefore, these color-coordinate variations scaled in terms of color-surround variations enable us to define a metameric index for diffuse reflectance materials that can now be compared for different pairs of illuminants in the same range:

$$M = \max_{ji}\{\Delta E_{ji},\ \forall j, i = 1 \ldots \alpha;\ \alpha \geq 2\}, \tag{9}$$

where M is the maximum value found for a pair of illuminants and α, the number of illuminants to be tested. Subscripts j,i represent different illuminants, with i fixed but arbitrary, whereas j could vary as much as the computer-software formulation or conventional light booth permits. It is worth noting that this type of index does not distinguish between reference and specimen panels, or between the first and second illuminant ($\Delta E_{ji} = \Delta E_{ij}$). In turn, the number of pairwise comparisons that would be made for a group of α illuminants are $\alpha!/[(\alpha - 2)!2!]$ (e.g., typical light booths contain 4 or 5 types of illuminants, so that the number comparisons of object pair-illuminants are 6 and 10, respectively).

Generalized tristimulus values would not be useful for examining local color differences at the specific aspecular angle γ, but their definition [see Eqs.(3) through (7)] would enable an overall measure of goniochromatism. Therefore, with Eqs. (3) and (4), it is straightforward to define a general metamerism index for spectral changes in illuminant as the maximum value calculated for a pair of illuminants (j,i):

$$\mathbf{M} = \max_{ji}\{\Delta\mathbf{E}_{ji},\ \forall j, i = 1 \ldots \alpha;\ \alpha \geq 2\} \tag{10}$$

$$\Delta\mathbf{E}_{ji} = \left[\sum_k \left(\frac{(\Delta\mathbf{T}_k)_j - (\Delta\mathbf{T}_k)_i}{\Delta'\mathbf{T}_k^S}\right)^2\right]^{1/2},$$

$$\text{where, } \Delta\mathbf{T}_k, \Delta'\mathbf{T}_k^S, \in [\Delta\mathbf{L}^*, \Delta\mathbf{a}^*, \Delta\mathbf{b}^*, \Delta\mathbf{C}^*, \Delta\mathbf{H}^*], \tag{11}$$

where

$$(\Delta\mathbf{T}_k)_j = \frac{\sum_\gamma \Phi_k(\gamma) \cdot (\Delta X_k(\gamma))_j \cdot \sin(\gamma) \cdot \Delta\gamma}{\sum_\gamma \Phi_k(\gamma) \cdot \sin(\gamma) \cdot \Delta\gamma}. \tag{12}$$

Note that $\Phi_k(\gamma)\ \forall k$ are assumed to be illuminant-independent; on the other hand, $\Delta'\mathbf{T}_k^S = \Delta' X_k^S, \forall k$. Eqs.(8) through (12) are based on the assumption that CIELAB is an approximately uniform color space.[23] When uniformity deviations are considered important, these circumstances can be taken into account in Eq. (8), normalizing each term of corrected color coordinates by suitable factors. Nevertheless, the same procedure in Eq. (11) would not be appropriate and goniochromatic functions could also prove tristimulus- dependent, so that the CIE limit [see Eq. (2)] would not be fulfilled. Eq. (11) does not involve a preferred aspecular-angle configuration between reference and specimen panels; however, due to the commutative property of addition, all possible configurations are equivalent to the canonical relative orientation, defined when reference and specimen tristimulus values are compared under the same aspecular angle γ [see Eq. (12)].

A Geometric Metamerism Index

Under a fixed illuminant and illuminating conditions, a pair of goniochromatic objects are called geometric metamers when, for specific aspecular angle, both samples are matched but color differences are produced when the aspecular angle is changed.[12] The coating industry has long recognized this as an undesirable effect in the color-reproduction process, because it involves changes in relative orientation of metal-flake and interference pigments with regard to reference and specimen panels. In addition, most coating finishes are designed to exhibit a maximum color variation, and marked directional effects could be achieved if pigment flakes were parallel to the surface support material. Otherwise, the more diffuse reflectance due to irregular pigment-flake orientation, the less the color variation. Colorists usually examine reference-specimen color differences by means of simultaneous color matching (i.e., gap time of zero), at each aspecular angle γ. Therefore, an overall geometric metamerism index can be defined for a pair of samples using the generalized tristimulus values as

$$\mathbf{GM} = \max_j\{\Delta\mathbf{E}_j^G,\ \forall j = 1 \ldots \alpha;\ \alpha \geq 1\} \tag{13}$$

$$\Delta\mathbf{E}_j^G = \left[\sum_k \left(\frac{(\Delta\mathbf{T}_k)_j}{(\mathbf{T}_k^S)_j}\right)^2\right]^{1/2},$$

where

$$\Delta\mathbf{T}_k \in [\Delta\mathbf{L}^*, \Delta\mathbf{a}^*, \Delta\mathbf{b}^*, \Delta\mathbf{C}^*, \Delta\mathbf{H}^*],$$

$$(\mathbf{T}_k^S)_j, \in [\mathbf{L}^*, \mathbf{a}^*, \mathbf{b}^*, \mathbf{C}^*, \mathbf{h}^*], \tag{14}$$

where $(\Delta\mathbf{T}_k)_j$ and $(\mathbf{T}_k^S)_j = (X_k^S)_j\ \forall k$, represents the kth color-coordinate variations between the reference and the specimen panels, and the kth color-surround coordinate under the jth illuminant, respectively, and α, the number of illuminants to be examined. Eq. (14) does not differentiate, either, between reference and specimen panels. Comments on uniform color spaces, canonical relative orientation, and illuminant independence on goniochromatic functions are the same as in the previous section.

A Flop Index

As an application of the last section, because generalized tristimulus values take into account the gonioappearance from a suitable geometric direction (the specular reflection for a fixed angle of illumination), a flop index could also be defined considering at least three viewing geometries—that

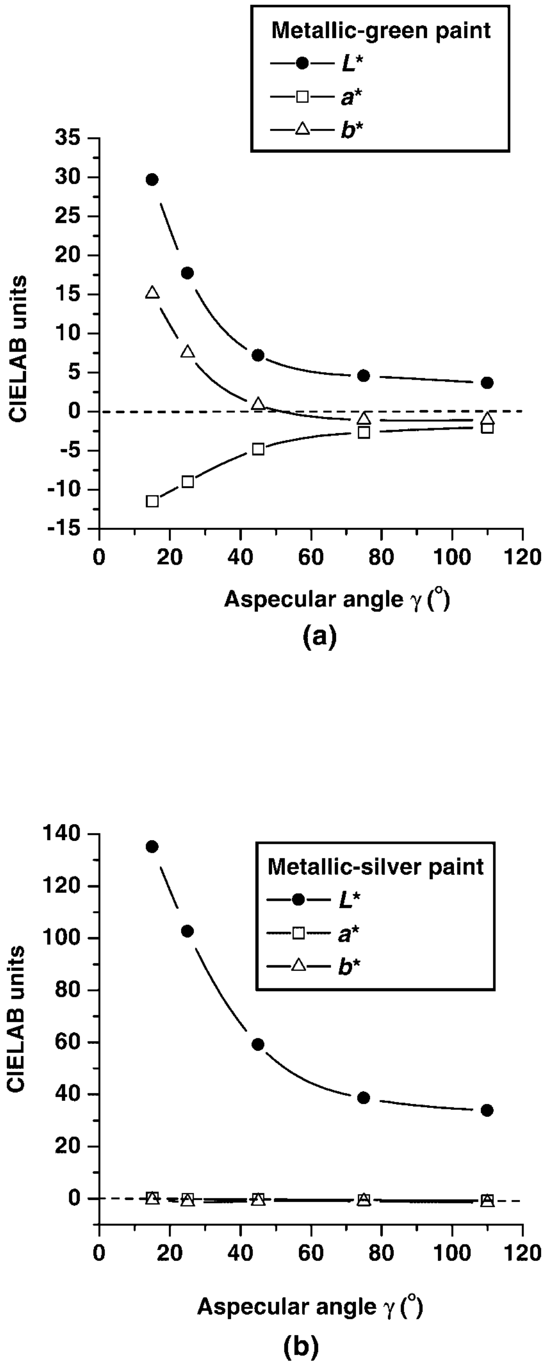

FIG. 2. CIE $L^*a^*b^*$ values measured with a multiangle spectrophotometer for different aspecular angles γ from typical (a) metallic-green and (b) metallic-silver paints.

is, under a fixed illuminant and illuminating conditions, when the viewing direction is changed from a near-specular to a flop angle crossing a face angle. For this, lightness and color flop of a goniochromatic specimen panel could be evaluated by eliminating generalized tristimulus values of the reference panel in Eq. (14). In turn, the corresponding flop index is the maximum presented for a variety of illuminants tested:

$$\mathbf{FI} = \max_j\{\Delta \mathbf{E}_j,\ \forall j = 1 \ldots \alpha;\ \alpha \geq 1\} \qquad (15)$$

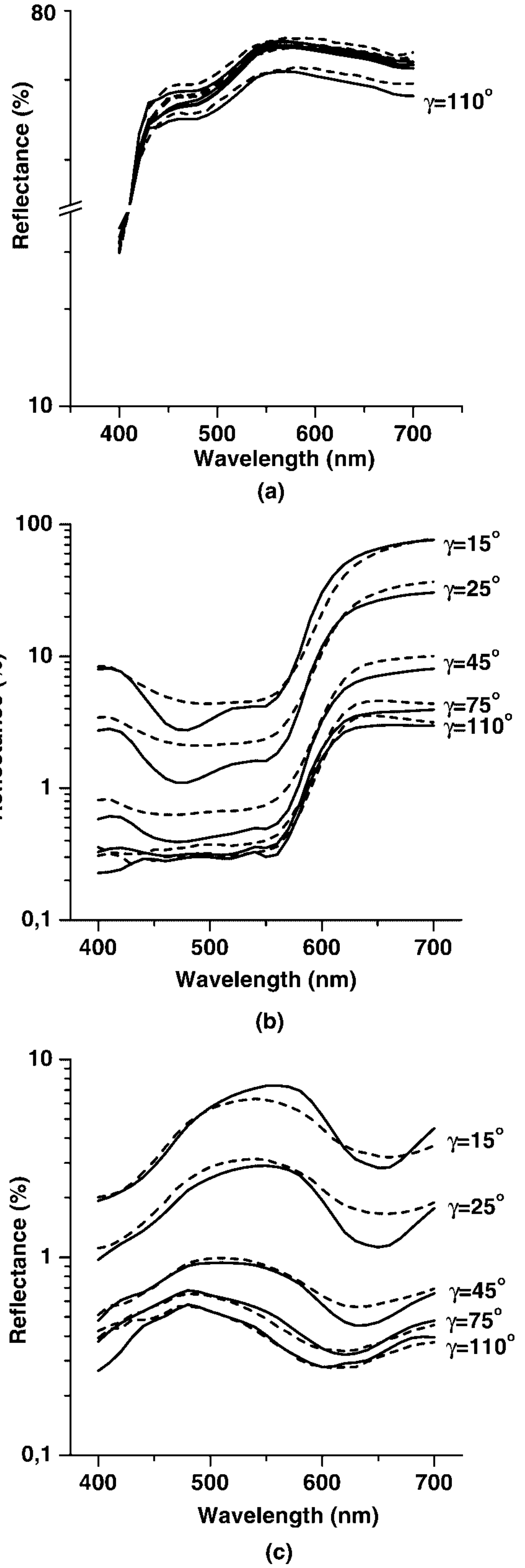

FIG. 3. Logarithm of the reflectance (%) for three matched pair samples at five different aspecular angles, corresponding to (a) a white–red–solid, (b) metallic-red, and (c) metallic-green color samples. Solid and dashed lines correspond to the reference and specimen panels, respectively.

TABLE I. The color coordinates of the three pair samples with the D65/10° standard observer in the CIELAB color space. color coordinates are expressed at each aspecular angle γ.

		Reference				Specimen			
Pair no.	γ(°)	L*(γ)	a*(γ)	b*(γ)	C*(γ)	L*(γ)	a*(γ)	b*(γ)	C*(γ)
1	15°	88.47	−0.58	6.56	6.58	88.48	−0.52	5.82	5.85
	25°	88.25	−0.60	6.54	6.56	88.30	−0.53	5.82	5.85
	45°	88.12	−0.58	6.35	6.38	88.40	−0.52	5.76	5.79
	75°	88.66	−0.57	5.31	5.34	88.98	−0.55	5.35	5.38
	110°	86.67	−0.55	4.87	4.90	86.96	−0.51	5.11	5.14
2	15°	42.15	51.60	29.73	59.55	39.93	45.51	20.07	49.74
	25°	26.64	38.43	23.33	44.96	28.34	36.19	18.14	40.48
	45°	11.89	24.14	13.47	27.64	14.08	24.22	13.99	27.97
	75°	7.80	18.60	8.45	20.43	8.62	19.47	9.61	21.71
	110°	6.63	16.44	7.05	17.89	6.97	17.48	7.43	19.00
3	15°	29.66	−11.48	15.07	18.94	27.85	−10.72	11.03	15.38
	25°	17.70	−8.99	7.50	11.71	18.80	−8.25	7.53	11.17
	45°	7.18	−4.80	0.83	4.88	7.56	−4.71	1.48	4.94
	75°	4.56	−2.67	−1.06	2.87	4.36	−2.30	−1.32	2.65
	110°	3.63	−2.00	−1.07	2.27	3.59	−1.73	−1.68	2.42

$$\Delta \mathbf{E}_j = \left[\sum_k \left(\frac{(\mathbf{T}_k)_j}{(\mathbf{T}_k^S)_j} \right)^2 \right]^{1/2},$$

$$\text{where } (\mathbf{T}_k)_j, (\mathbf{T}_k^S)_j \in [\mathbf{L}^*, \mathbf{a}^*, \mathbf{b}^*, \mathbf{C}^*, \mathbf{h}^*], \quad (16)$$

where j also varies throughout the illuminant group examined ($j = 1,2, \ldots \alpha$; $\alpha \equiv$ number of illuminants).

DISCUSSION

Estimating Goniochromatic Discrimination Functions

The generalized illuminant metamerism, the geometric metamerism, or the flop index [Eqs. (10), (11), and (13) through (16)] are not possible if $\Phi_k(\gamma)$ values are not de-

TABLE II. Flop index (**FI**) for each member of the three pair samples tested under eight illuminants. Goniochromatic discrimination functions were approximated considering (a) BASF coefficients and (b) BASF and DIN 6175-2 coefficients. **FI** values represent the maximum values calculated for the eight illuminants tested.

		BASF		BASF & DIN 6175-2	
Pair no.	Illuminant	Reference	Specimen	Reference	Specimen
1	D65	87.59	87.88	379.64	380.26
	A	87.89	88.19	380.62	381.08
	F2	88.01	88.25	383.16	383.16
	F11	87.98	88.22	381.37	381.05
	D50	87.67	87.96	379.40	379.84
	D75	87.55	87.84	379.79	380.54
	C	87.99	88.26	378.98	379.74
	F7	88.07	88.33	378.86	379.25
	FI	88.01	88.25	383.16	383.16
2	D65	23.60	24.40	494.75	508.65
	A	27.69	28.87	569.33	590.56
	F2	22.54	22.44	422.14	423.24
	F11	26.01	26.44	534.71	540.12
	D50	25.31	26.22	530.16	546.27
	D75	22.75	23.51	476.81	489.85
	C	24.30	25.28	484.10	498.53
	F7	24.07	24.93	479.75	491.52
	FI	27.69	28.87	569.33	590.56
3	D65	5.71	5.54	83.68	78.54
	A	5.37	5.26	80.26	77.16
	F2	5.26	5.12	67.89	66.12
	F11	5.44	5.24	80.46	74.90
	D50	5.65	5.50	83.50	79.09
	D75	5.74	5.56	83.91	78.37
	C	7.39	7.29	82.27	77.05
	F7	7.34	7.26	80.39	76.17
	FI	5.71	5.54	83.68	79.09

termined. For this, the type of pigments added in a basecoat are important. Figure 2 represents CIELAB color coordinates measured with the D65/10° standard observer from a typical automotive metallic-green paint (carbon-black, phthalocyanine blue and green, green-interference and lenticular aluminum-flake pigments) and metallic-silver paint (only aluminum corn-flake pigments) at different aspecular angles γ. Color coordinates were connected by polynomial functions.[12]

Metal flakes are responsible basically for changes in lightness [see Fig. 2(b)] and interference pigments for changes in chromaticity [see Fig. 2(a)]. Relative lightness variations are the most important, whereas color variations (a^*, b^* changes) are second-order effects. Unfortunately, there is no accepted procedure for establishing human standard $\Phi_i(\gamma)$ functions from situations such as those in Fig. 2 for the 2° and 10° CIE standard observers. Nevertheless, we could make a preliminary estimation by considering the works of Schmittmann and Cloppenburg[24] and the DIN 6175-2.[15] Under the assumption that the CIE (1976) $L^*a^*b^*$ is an approximately uniform color space, one possible way to estimate $\Phi_i(\gamma)$ would be by taking these as the BASF D^2-value coefficients.[24] The D^2 value is a total color-difference formula that generalizes the CIELAB formula for goniochromatic objects, adding weighted color-coordinate differences at each aspecular angle γ. These weighting factors associated at each color-coordinate variation were established on correlations with goniochromatic visual experiences. At flop angles visual luminance differences would be greater than at face angles, and these, greater than at near-specular angles, chromatic differences would in general be greater than luminance at all aspecular angles. For saturated colors, BASF factors also provide chroma- and hue-difference weighting factors. Chroma weighting factors were identical to luminance factors, because neither dynamics were clearly distinguishable. On the other hand, red–green, yellow–blue variations or hue differences were greater, but aspecular angle dependencies were not found. As a numerical example, three matched pair samples were used to calculate generalized tristimulus values, under BASF approximation. Figure 3 represents the logarithm of the reflectance for the reference and the specimen panels at five aspecular angles (15°, 25°, 45°, 75°, and 110°), whereas Table I, shows the CIELAB tristimulus values measured at each aspecular angle with the D65/10° standard observer.

Pair 1 constitutes a conventional white–red solid-color pair sample (titanium dioxide and opaque red pigments), whereas pair 2 represents their goniochromatic counterpart, which enables the comparison of the magnitudes of goniochromatic effects (aluminum metal-flake and transparent-red pigments). Finally, pair 3 is the green-metallic sample mentioned above, which enables comparisons of the magnitudes between different goniochromatic pair samples. Ignoring color-surround effects, Tables II, III, and IV (the third and fourth columns, the third column, and the left inferior triangles, respectively) present a numerical estimation for the flop, the geometric, and the generalized LABD metamerism index for changes in spectral composition of illuminants for the three matched pairs under eight different illuminants (D65, A, F2, F11, D50, D75, C, F7)[†].

Despite the fact that these weighting factors take into account the basic human goniochromatic sensitivity for

TABLE III. Geometric metamerism (**GM**) index of the three pair samples tested under eight illuminants. Goniochromatic discrimination functions were approximated considering (a) BASF coefficients and (b) BASF and DIN 6175-2 coefficients. **GM** values represent the maximum values calculated for the eight illuminants tested.

Pair no.	Illuminant	BASF	BASF & DIN 6175-2
1	D65	0.30	1.09
	A	0.36	4.57
	F2	0.42	3.60
	F11	0.44	7.97
	D50	0.31	2.33
	D75	0.29	0.78
	C	0.27	1.00
	F7	0.27	1.75
	GM	0.44	7.97
2	D65	1.09	5.39
	A	1.43	6.38
	F2	0.57	3.55
	F11	0.78	4.29
	D50	1.18	5.58
	D75	1.05	5.31
	C	0.93	5.54
	F7	0.87	5.04
	GM	1.43	6.38
3	D65	0.46	10.26
	A	0.41	9.10
	F2	0.21	9.69
	F11	0.50	10.99
	D50	0.44	9.67
	D75	0.48	10.56
	C	0.47	10.44
	F7	0.43	9.63
	GM	0.50	10.99

[†] A distinction should be made to use the corresponding weighting factors when goniochromatic objects are chromatic or achromatic. Under the two approximations made, the flop index was evaluated using the corresponding L^*,a^*,b^* weighting factors due to the fact that BASF approximation does not provide absolute factors for C^* and h^*, but ΔC^* and ΔH^*. An obvious generalization to treat the situation on geometric metamerism and on the generalized illuminant metamerism index might be to impose the chromatic or achromatic and pastel DIN 6175-2 criterion at each aspecular angle to distinguish goniochromatic objects at different color areas. The problem emerges when both criteria are satisfied at different aspecular angles. In principle, it is not possible to apply the DIN 6175-2 or BASF connecting functions, due to the fact that both are not oriented to calculate differences between generalized tristimulus values under different illuminants. An alternative criterion might be to evaluate the DIN 6175-2 criterion at three aspecular angles that characterize goniochromatism for each matched sample (e.g., 15°, 45°, and 110°). If two of the three viewing geometries accept the chromatic criterion, then samples are evaluated with the use of the corresponding chroma- and hue-difference goniochromatic functions; otherwise, red–green and yellow–blue goniochromatic functions are applied. Under this criterion, pairs 1, 2, and 3 were evaluated as achromatic, chromatic, and achromatic, respectively.

TABLE IV. Generalized metamerism index for spectral change in illuminant for the three pair samples tested under eight illuminants. Goniochromatic discrimination functions were approximated considering (a) BASF coefficients (left lower triangle), and (b) BASF and DIN 6175-2 coefficients (right upper triangle). right top and left bottom **GM** values represent the maximum values calculated for the eight illuminants tested under DIN 6175-2 and BASF, and BASF only, respectively.

Pair no. 1	D65	A	F2	F11	D50	D75	C	F7	**GM**
D65		3.70	2.80	7.15	1.44	0.69	0.48	0.91	7.84
A	0.17		1.50	3.49	2.33	4.37	4.02	3.03	
F2	0.31	0.26		4.53	1.39	3.48	3.23	1.94	
F11	0.33	0.17	0.27		5.73	7.84	7.50	6.40	
D50	0.06	0.11	0.27	0.26		2.13	1.84	0.71	
D75	0.03	0.20	0.34	0.36	0.10		0.43	1.56	
C	0.03	0.18	0.33	0.34	0.09	0.03		1.38	
F7	0.05	0.14	0.27	0.29	0.05	0.08	0.06		
M	0.36								

Pair no. 2	D65	A	F2	F11	D50	D75	C	F7	**G**
D65		2.72	2.61	1.57	0.48	0.13	0.32	0.42	2.98
A	0.39		2.88	2.38	2.25	2.84	2.98	2.58	
F2	0.62	0.95		1.05	2.55	2.61	2.90	2.19	
F11	0.38	0.70	0.25		1.50	1.58	1.87	1.16	
D50	0.11	0.29	0.71	0.46		0.61	0.73	0.58	
D75	0.04	0.43	0.58	0.34	0.15		0.29	0.43	
C	0.71	1.00	0.47	0.47	0.78	0.68		0.72	
F7	0.83	1.13	0.46	0.54	0.91	0.80	0.15		
M	1.13								

Pair no. 3	D65	A	F2	F11	D50	D75	C	F7	**M**
D65		1.40	1.86	2.05	0.60	0.31	0.20	0.76	3.89
A	0.063		2.49	2.14	0.90	1.63	1.51	1.26	
F2	0.294	0.234		3.89	1.77	2.06	2.03	1.31	
F11	0.093	0.098	0.306		2.26	1.92	1.91	2.66	
D50	0.027	0.041	0.267	0.102		0.89	0.77	0.46	
D75	0.014	0.074	0.306	0.088	0.040		0.13	1.06	
C	0.052	0.083	0.308	0.109	0.063	0.051		0.96	
F7	0.054	0.069	0.278	0.132	0.047	0.063	0.045		
M	0.308								

object-color stimuli, more development is needed. It is worth noting that although a greater aspecular angle dependency on the spectral reflectance is found for pairs 2 and 3 (see Fig. 3), the flop indices are not appropriately scaled. The BASF coefficients grow as the specular angle does, but weighting factors are not adequate to bring about a direct correlation to compensate for the higher L^* values found for pair 1 (see Table I).

Relative tristimulus variations between matched reference and specimen samples correct high L^* values at each aspecular angle; therefore, pair 2 presents a higher geometric metamerism value than that of pair 1, whereas the geometric metamerism value of pair 3 has a value that is lower value than that of pair 2 but higher than that of pair 1. Finally, a generalized metamerism index for changes in spectral composition of illuminants of pair 2 is also higher than that of pair 1, whereas that of pair 3 is lower than that of pair 2, and slightly lower than that of pair 1.

It is notable that BASF values are quite old (Cloppenburg, personal communication, 2001). These coefficients lie on the aspecular angle $\gamma = 25°$ baseline values. Schmittmann and Cloppenburg[24] indicated that these coefficients contain pragmatic values, and the baseline can be changed while maintaining the gonioappearance (i.e., more sensitivity in flop than in face and near-specular angles); therefore, more suitable baseline values would be better to determine the scale of human goniochromatic sensitivity according to visual experience. At the same time, BASF coefficients show no differences between redness–greenness and yellowness–blueness goniochromatic factors. These results are not compatible with many psychophysical studies that have reported different sensitivities on red–green and yellow–blue channels in tasks such as chromatic discrimination[25] or spatial vision.[26] Different color-opponent values would involve a better chroma-geometric sensitivity and therefore a better goniodifference between chroma and luminance.

A more elaborate approximation might consist of taking Φ_i at the expense of ignoring the CIE limit [see Eq. (2)]. Therefore, goniochromatic discrimination functions $\Phi_i = \Phi_i(\gamma, X_i)$ could be made as the product of two contributions, which, expressed as normalizing factors, could be written as

$$\Phi_i(\gamma, X_i) = \frac{1}{g_i(\gamma) \cdot \xi_i[X_i(\gamma)]}, \quad i \in [L^*, a^*, b^*, C^*, h^*], \tag{17}$$

where $g_i(\gamma)$ represents the goniochromatic factor associated with the ith color-coordinate variation at the specular angle

γ, and $\xi_i[X_i(\gamma)]$ the tristimulus-value dependency at the same angle γ. To date, the best values for $\xi_i[X_i(\gamma)]$ are the DIN 6175-2[15] coefficients. Tables II, III, and IV (the fifth and sixth columns, the fourth column, and the right superior triangles, respectively) also present a numerical estimation for the flop, the geometric and the generalized LABD metamerism index for changes in spectral composition of illuminants for the three matched pairs, taking into account BASF and the DIN 6175-2 coefficients simultaneously. Differences between the two approximations are found. Numerical values are higher because generalized tristimulus values were calculated such that tristimulus constants (k_i') were the unity. On the other hand, the maximum values calculated for the generalized illuminant metamerism index differ under the two approximations in goniochromatic samples (see Table IV). The same conclusion follows for the flop index for pair 3 (see Table II).

Under this estimation, the flop index between pairs1 and 2 are now directly related. This could reside in the chromatic dependency: Redness–greenness and yellowness–blueness goniochromatic factors in pair 1 are higher than face angles and lightness goniochromatic factors are lower than flop angles with respect to the first approximation (BASF coefficients only), whereas, in pair 2, redness–greenness, yellowness–blueness, and lightness goniochromatic factors are higher than face and flop angles, respectively. Nevertheless, these chromatic dependencies on $\Phi_i(\gamma,X_i)$ functions are not sufficient, because the flop index in pair 3 is still less than that in pair 1. Moreover, geometric metamerism of pair 2 now has the least value due to the low values reached by the goniochromatic function associated with chroma in respect to the value obtained using BASF coefficients only. The generalized metamerism index for spectral changes in illuminant again shows an inverse relation between pairs 1 and 2, and between pairs 2 and 3, with respect to the first estimation.

CONCLUSIONS

A generalization of a metameric index for changes in spectral composition of illuminants has been defined for goniochromatic objects over a limited number of pairwise illuminant comparisons. Metameric differences could be scaled in terms of color-surround changes to treat illuminant influence. Consequently, a geometric metamerism and a flop index that take into account goniochromism and luster are also defined. These expressions could include uniformly colored objects as the limit case and would be useful in computer color-matching and color-quality control over a wide range of industrial-coating applications, especially automotive paints, where attractive goniochromatic effects are important. Nevertheless, their numerical and practical implementation depends on the definition of a standard method to determine the goniochromatic discrimination functions for the CIE standard observers.

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